Recasting the Model in Terms of Visits

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Recasting the model to focus on visits to shops

Household i chooses visits v_i to max. utility s.t. budget constraint, taking as given tightness x & price p.

Household's problem

$q(x),\ v$

- consumption: $c_i = y_i - p \cdot v_i = [q(x) - p] \cdot v_i$
- spending/output: $y_i = q(x) \cdot v_i$

$$\max_{v_i} \frac{x}{1+x}\left[(q(x)-p)\, v_i\right]^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x}\left(\frac{m_i}{p}\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

$$\text{s.t.} \quad m_i + p \cdot q(x)\, v_i = \mu_i + p\, f(x)\, k_i$$

m_i

$$\hookrightarrow \max_{v_i} \frac{x}{1+x}\left[(q(x)-p)\, v_i\right]^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x}\left[-q(x)\, v_i + \frac{\mu_i}{p} + f(x) k_i\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

concave maximization problem

FOC:

$$\frac{x^{-\varepsilon}}{\cancel{1+x}} \cdot \cancel{\frac{\varepsilon-1}{\varepsilon}}\, [q(x)-p]^{\frac{\varepsilon-1}{\cancel{\varepsilon}}}\, v_i^{\cancel{-1/\varepsilon}} = \frac{q(x)^{-\varepsilon}}{\cancel{1+x}}\, \cancel{\frac{\varepsilon-1}{\varepsilon}} \left[\quad\right]^{\cancel{-1/\varepsilon}} \cancel{= 0}$$

$$v_i = \frac{x^{\varepsilon}}{q(x)^{\varepsilon}}\, [q(x)-p]^{\varepsilon-1} \left[f(x) k_i + \frac{\mu_i}{p} - q(x)\, v_i\right]$$

$$\left[1 + x^{\varepsilon} q(x)^{1-\varepsilon}\, [q(x)-p]^{\varepsilon-1}\right] \cdot v_i = \frac{x^{\varepsilon}}{q(x)^{\varepsilon}}\, [q(x)-p]^{\varepsilon-1} \left[f(x) k_i + \frac{\mu_i}{p}\right]$$

$$v_i(x,p) = \frac{x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{-\varepsilon}}{1 + x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}} \left[j(x) k_i + \frac{w_i}{p} \right]$$

$$q(x)\, v_i(x,p) = \frac{x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}}{1 + x^{\varepsilon}(q(x)-p)^{\varepsilon-1} q(x)^{1-\varepsilon}} \left[j(x) k_i + \frac{w_i}{p} \right]$$

$y_i(x,p)$

$\sigma(x) = \text{MPS}$

[initial wealth + income]

<u>Aggregate number of visits</u> $v(x,p) = \sum_i v_i(x,p)$

$$q(x)\, v(x,p) = \sigma(x) \left[j(x) \cdot k + \frac{w}{p} \right]$$

same as AD curve