

# **Consumption and Saving in the Heterogeneous-Agent Model**

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Pascal Michailat  
<https://pascalmichailat.org/c2/>

Household  $i$  consumes  $c_i$  services & holds  $m_i$  units of money. Household  $i$  takes market tightness  $x$  and price of services  $p$  as given.

Household  $i$  maximize utility subject to budget constraint

$$\max_{c_i, m_i} \frac{x}{1+x} c_i^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x} \left( \frac{m_i}{p} \right)^{\frac{\varepsilon-1}{\varepsilon}} \quad (\varepsilon > 1) \quad (x > 0)$$

$$\text{st } p \cdot [1 + \tau(x)] c_i + m_i = p \cdot f(x) \cdot \underline{k}_i + \underline{\nu}_i$$

$$\max_{c_i} \frac{x}{1+x} c_i^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x} \left[ f(x) \underline{k}_i + \frac{\underline{\nu}_i}{p} - [1 + \tau(x)] c_i \right]^{\frac{\varepsilon-1}{\varepsilon}}$$

$$\text{FOC: } c_i^{-\frac{1}{\varepsilon}} = \frac{1 + \tau(x)}{x} \left[ f(x) \underline{k}_i + \frac{\underline{\nu}_i}{p} - (1 + \tau(x)) c_i \right]^{-\frac{1}{\varepsilon}}$$

$$c_i = \left[ \frac{x}{1 + \tau(x)} \right]^{\varepsilon} \left[ f(x) \underline{k}_i + \frac{\underline{\nu}_i}{p} - (1 + \tau(x)) c_i \right]$$

$$\left[ 1 + x^{\varepsilon} [1 + \tau(x)]^{1-\varepsilon} \right] c_i = x^{\varepsilon} [1 + \tau(x)]^{-\varepsilon} \left[ f(x) \underline{k}_i + \frac{\underline{\nu}_i}{p} \right]$$

$$c_i = \frac{x^{\varepsilon} [1 + \tau(x)]^{-\varepsilon}}{1 + x^{\varepsilon} [1 + \tau(x)]^{1-\varepsilon}} \cdot \left[ f(x) \underline{k}_i + \frac{\underline{\nu}_i}{p} \right]$$

↑ consumption of services

↑  $\varepsilon \in (0, 1)$

↑ total real wealth, before spending (endowment + income)

$$y_i = [1 + z(x)] c_i = \frac{x^\varepsilon [1 + z(x)]^{1-\varepsilon}}{1 + x^\varepsilon [1 + z(x)]^{1-\varepsilon}} \left[ f(x) k_i + \frac{\mu_i}{p} \right]$$

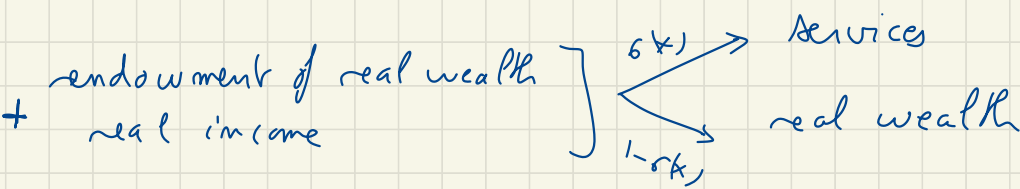
↑ purchases of services  $\sigma(x) \in (0, 1)$  ↑ initial real wealth

$$y_i = \sigma(x) \cdot \left[ f(x) k_i + \frac{\mu_i}{p} \right]$$

$$\frac{m_i}{p} = f(x) k_i + \frac{\mu_i}{p} - \underbrace{[1 + z(x)] c_i}_{y_i = \text{purchases} = \sigma(x) \left[ f(x) k_i + \frac{\mu_i}{p} \right]}$$

↑ savings = real wealth

$$\frac{m_i}{p} = [1 - \sigma(x)] \left[ f(x) k_i + \frac{\mu_i}{p} \right]$$



$$v_i = \frac{y_i}{q(x)} = \frac{\sigma(x)}{q(x)} \left[ f(x) k_i + \frac{\mu_i}{p} \right] - v_i$$

↑ vicks

key function:  $\sigma(x) \in (0, 1)$  is the Marginal Propensity to Spend (MPS)

→ marginal propensity to spend out of  
wealth & income

→  $[1 - \sigma(x)]$  is the marginal propensity to  
save, also  $\in (0, 1)$