Consumption and Saving in the Heterogeneous-Agent Model

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Household i consumes c_i services & holds m_i units of money. Household i takes market tightness x and price of services p as given.

Household i maximize utility subject to budget constraint

$$\max_{c_i, m_i} \quad \frac{x}{1+x} c_i^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x}\left(\frac{m_i}{p}\right)^{\frac{\varepsilon-1}{\varepsilon}} \qquad \begin{matrix}(\varepsilon>1)\\ (x>0)\end{matrix}$$

$$\text{st} \quad p \cdot [1+\tau(x)]\, c_i + m_i = p \cdot f(x) \cdot k_i + \nu_i$$

$$\max_{c_i} \quad \frac{x}{1+x} c_i^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+x}\left[f(x)\, k_i + \frac{\nu_i}{p} - [1+\tau(x)]\, c_i\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

$$\underline{\text{FOC}}: \quad c_i^{-1/\varepsilon} = \frac{1+\tau(x)}{x}\left[f(x)\, k_i + \frac{\nu_i}{p} - (1+\tau(x))\, c_i\right]^{-1/\varepsilon}$$

$$c_i = \left[\frac{x}{1+\tau(x)}\right]^{\varepsilon}\left[f(x)\, k_i - \frac{\nu_i}{p} - (1+\tau(x))\, c_i\right]$$

$$\left[1 + x^{\varepsilon}[1+\tau(x)]^{1-\varepsilon}\right] c_i = x^{\varepsilon}[1+\tau(x)]^{-\varepsilon}\left[f(x)\, k_i + \frac{\nu_i}{p}\right]$$

$$c_i = \frac{x^{\varepsilon}[1+\tau(x)]^{-\varepsilon}}{1+x^{\varepsilon}[1+\tau(x)]^{1-\varepsilon}} \cdot \left[f(x)\, k_i + \frac{\nu_i}{p}\right]$$

consumption of services

$\in (0,1)$

total real wealth, before spending (endowment + income)

$$y_i = [1+\tau(x)]\, c_i = \frac{x^{\varepsilon}[1+\tau(x)]^{1-\varepsilon}}{1+x^{\varepsilon}[1+\tau(x)]^{1-\varepsilon}} \left[f(x) k_i + \frac{\mu_i}{p} \right]$$

purchase of services

$\sigma(x) \in (0,1)$

initial real wealth

$$\boxed{y_i = \sigma(x) \cdot \left[f(x) k_i + \frac{\mu_i}{p} \right]}$$

$$\frac{m_i}{p} = f(x) k_i + \frac{\mu_i}{p} - \underbrace{[1+\tau(x)] c_i}_{\substack{y_i = \text{purchases} \\ = \sigma(x)\left[f(x) k_i + \frac{\mu_i}{p}\right]}}$$

savings = real wealth

$$\boxed{\frac{m_i}{p} = [1-\sigma(x)] \left[f(x) k_i - \frac{\mu_i}{p} \right]}$$

$$\left.\begin{array}{l} \text{endowment of real wealth} \\ + \text{ real income} \end{array}\right] \begin{array}{l} \xrightarrow{\sigma(x)} \text{services} \\ \xrightarrow{1-\sigma(x)} \text{real wealth} \end{array}$$

$$v_i = \frac{y_i}{q(x)} = \boxed{\frac{\sigma(x)}{q(x)} \left[f(x) k_i + \frac{\mu_i}{p} \right] = v_i}$$

visits

<u>Key function:</u> $\sigma(x) \in (0,1)$ is the

<u>Marginal Propensity to Spend (MPS)</u>

→ marginal propensity to spend out of wealth & income

→ $[1 - \sigma(x)]$ is the marginal propensity to save, also $\in (0, 1)$