Solving the Household's Problem

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Household's problem (concave maximization problem):

$$\max_{c} \quad \frac{X}{1+X}\, c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+X}\left[\frac{w}{p} + \rho \cdot k - (1+\tau)\, c\right]^{\frac{\varepsilon-1}{\varepsilon}}$$

Necessary & sufficient condition for the optimal c (c maximizing utility): derivative = 0

Here:

$$\frac{X}{\cancel{1+X}} \cdot \cancel{\frac{\varepsilon-1}{\varepsilon}} \cdot c^{-\frac{1}{\varepsilon}} - [1+\tau(x)]\,\frac{1}{\cancel{1+x}}\,\cancel{\frac{\varepsilon-1}{\varepsilon}}\,\Big[\;\underbrace{\dots}_{\cancel{\frac{\mu}{p}} + \int k - (1+\tau)c \,=\, \frac{m}{p}}\;\Big]^{-1/\varepsilon} = 0$$

$$\underbrace{X\,c^{-1/\varepsilon}}_{\propto \text{ MU of services}} = \underbrace{[1+\tau(x)]}_{\substack{\text{price of services relative}\\ \text{to real money balances}}}\;\underbrace{[m/p]^{-1/\varepsilon}}_{\substack{\propto \text{ MU of real money}\\ \text{balances / real wealth}}}$$

$$c^{-1/\varepsilon} = \frac{1+\tau(x)}{X}\,(m/p)^{-1/\varepsilon}$$

$$\boxed{c = \left[\frac{X}{1+\tau(X)}\right]^{\varepsilon} \cdot \frac{m}{p}}$$

- c : consumption of service
- # services purchased by household : y

$$\boxed{y = [1+\tau(x)] \cdot c}$$

- # visits by household : v

$$\boxed{v = y/q(x) = c\,[1+\tau(x)]/q(x)}$$