Urn-Ball Matching Function

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Market with B buyers & S sellers

Example. restaurants & lunch customers

$\quad$ S restaurants $\quad$ (one table, one meal)

$\quad$ B lunch customers $\quad$ (one meal at lunch)

__One restaurant.__ $\quad \dfrac{1}{5} \quad \cdot \quad$ proba to get one specific customer

$\qquad 1 - \frac{1}{5} \quad \cdot \quad$ proba to NOT get customer

$\left(1 - \dfrac{1}{5}\right)^B \quad \cdot \quad$ proba to get NO customer

$1 - \left(1 - \frac{1}{5}\right)^B = \quad$ proba to get AT LEAST ONE customer

$\qquad = \quad$ proba to sell a meal

$\qquad = \quad$ expected # meals sold by one restaurant

__Expected # meals sold by all restaurants__

$\qquad S \times \left[1 - \left(1 - \dfrac{1}{5} \right)^B \right] \quad = \quad$ # trades / meals sold

__Simplification__ $\quad \left(1 - \dfrac{1}{5}\right)^B = \exp\left(B \cdot \ln\left(1 - \dfrac{1}{5} \right) \right)$

$\qquad \ln(1 - x) \sim -x \qquad$ when $x \sim 0$

$$\left(1 - \frac{1}{s}\right)^B \sim \exp\left(B \times -(1/s)\right)$$

$$\sim \exp\left(-B/s\right)$$

$$\# \text{ meals} = \boxed{\underline{\underline{s}} \times \left(1 - \exp\left(-B/s\right)\right) = M(s, B)}$$

um-ball   matching function (clearly <u>CRS</u>)

$$- \quad M(0, B) = 0 \qquad M(s, 0) = 0$$

$$- \quad \frac{\partial M}{\partial B} > 0 \qquad\qquad - \frac{\partial M}{\partial s} \to \text{ need to check}$$

$$\frac{\partial M}{\partial s} = 1 - \exp\left(-B/s\right) + \left[-\frac{B}{s^2}\exp\left(-\frac{B}{s}\right)\right] s$$

$$= 1 - e^{-B/s} - \frac{B}{s} e^{-B/s}$$

$$= 1 - \left[1 + \frac{B}{s}\right] e^{-B/s}$$

$$\frac{\partial M}{\partial s} = 1 - \frac{1 + B/s}{\exp(B/s)}$$

less than!

$$\boxed{e^x > 1 + x}$$

$$\frac{\partial M}{\partial s} > 0$$